Forbidden Transitions in Muonic ^{208}Pb*

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ABSTRACT

Electric quadrupole ($4f \rightarrow 2p$ and $3d \rightarrow 1s$) atomic transitions have been observed in muonic ^{208}Pb. An identification based upon energy and relative intensity determinations provides good agreement with theoretical predictions.

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An atomic transition which violates the electric dipole (E1) selection rule is considered a "forbidden transition" because its transition rate is generally much smaller than the rate of the competing dipole transition. The first forbidden transitions, recognized as such, were observed by Datta[1] in 1922 from electronic atoms of alkali metals. Since then many forbidden transitions in electronic atoms have been observed. The subject has been reviewed by Garstang[2]. Until now[3] only E1 muonic transitions have been reported. We would like to report the observation of some electric quadrupole (E2) atomic transitions in muonic ^{208}Pb.

In the long wave length approximation the probability of a bound muon making an Lth order electric (EL) atomic transition is given by:

$$\underset{j\to j'}{P(EL)} = 2\alpha \frac{mc^2}{\hbar} \frac{(L+1)}{(2L+1)L[(2L+1)!!]^2} \left[\frac{\Delta E}{mc^2}\right]^{2L+1} (2j'+1) \begin{pmatrix} j & j' & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \frac{1}{(\alpha Z)^{2L}}$$

$$\times \left[\frac{Z^L}{a_\mu} \int_0^\infty r^L \, (ff' + gg') \, r^2 dr\right]^2 \qquad (1)$$

where f and g (f' and g') are the initial (final) radial Dirac wave functions for a muon bound to a nucleus with charge Z;

a_μ is the muon Bohr radius;

ΔE is the energy of a muonic transition between atomic states with total angular momenta j and j'.

E2 transitions can be observed with moderate ease in muonic atoms of high Z when it is possible for a muon to make an energetically favored E2 transition with a change in principal quantum number $\Delta n > 1$ instead of an E1 transition with $\Delta n = 1$. For example, the calculated value of the ratio R, of deexcitation intensities from the 3d atomic state via E1 and E2 transitions, is given (using nonrelativistic wave functions for a point nuclear charge distribution) by:

$$R(E1/E2) = \frac{P(3d \to 2p)}{P(3d \to 1s)} = \frac{15}{(Z\alpha)^2}$$

$$= 42 \text{ for } {}^{208}\text{Pb} \qquad (2)$$

Using relativistic wave functions for a finite distribution of nuclear charge we find that the predicted ratio R decreases by more than a factor of 2 to:

$$R = 20.4 \text{ for } {}^{208}\text{Pb}$$

Indeed, such E2 transitions from circular orbits ($l = n-1$) can be of comparable intensity to E1 transitions from inner orbits ($l < n-1$) which are much less populated than the circular orbits.

With the muon fluxes obtainable from the 600 MeV synchrocyclotron at the NASA Space Radiation Effects Laboratory, E2 transitions from circular orbits are easily observed as long as they can be resolved from E1 transitions of nearly the same energy. For example the $3p \to 1s$ E1 (K_β) transitions lie close to the $3d \to 1s$ E2 transitions in ^{208}Pb with energies around 8.5 MeV. In muonic Pb the finite size effects and radiative corrections in the n = 3 level produce a displacement of the 3d and 3p doublets by about 40 keV, which is easily resolved by Ge(Li) detectors. Similarly the $4f \to 2p$ triplet is displaced from the $4d \to 2p$ triplet by 17 keV, again readily resolved. However for lower Z elements such displacements would be less. In addition the fine-structure splitting is generally of the same order of magnitude as the E1-E2 multiplet displacements so that certain components of the multiplets do coincide in energy (e.g. the $4d_{5/2} \to 2p_{3/2}$ and the $4f_{5/2} \to 2p_{3/2}$ transitions in ^{208}Pb differ by only 0.5 keV). Since the population of inner orbits in high Z muonic atoms is experimentally[4] less than that of the circular orbits, these transitions can be of comparable intensity as our ^{208}Pb data demonstrate. Obviously unresolved E1 and E2 transitions may lead to faulty interpretations in terms of charge distributions if the observed energy is shifted and if such unresolved transitions are taken as pure E1 transitions.

The near coincidence between certain E2 transitions from circular orbits with E1 transitions from inner orbits of the same principal quantum number does allow a convenient method of studying the atomic cascade since one can infer the relative population of states of different orbital angular momentum with essentially no correction for detector efficiency or self-absorption in the target. In order to be able to infer the relative populations it is necessary that the transition rates be calculable. For atomic transitions where the muon wave functions are well-known, such calculations, in principle, present no insuperable problem. It must be stressed, though, that for high Z muonic atoms, the long wave length approximation formulas used in this discussion should be replaced by the exact expression[5] containing spherical Bessel functions resulting from the multipole expansion of the vector potential.

In Fig. 1 we show the muonic x-ray spectrum -- both prompt and delayed -- in the region of the K_β transitions in ^{208}Pb. The $3d \to 1s$ doublets are quite well resolved. The solid curve indicates a four Gaussian least squares fit to the data with an assumed exponential background which is indicated by the dashed line below. The line spectrum beneath the data indicates the positions of the four peaks. We base our identification of the E2 transitions on the observed energies and on the relative intensities, which are for the most part in good agreement with the predicted energies and intensities. The predicted energies were determined by a 3-parameter fit of our observed energies[6] for the $3d_{3/2} \to 2p_{1/2}$, $3d_{5/2} \to 2p_{3/2}$, $2p_{3/2} \to 1s_{1/2}$, $2p_{1/2} \to 1s_{1/2}$, $2s_{1/2} \to 2p_{1/2}$ and $2s_{1/2} \to 2p_{3/2}$ transitions to a 2-parameter Fermi distribution and a variable nuclear polarization of the 1s level. We have included the corrections of Chen[7] for nuclear polarization in the higher levels and the higher order vacuum polarization corrections of Fricke[8]. A complete discussion of this analysis will appear elsewhere[9]. The values[10] obtained from this fit are given in Table I.

For the purpose of identification the transition rates were calculated using the long wave length approximation given in Eq(1). The cascade calculations[11] were made assuming a statistical distribution in the n = 14 level. Muon wave functions for a point charge distribution were used for all transitions feeding the n = 4 levels and for all $\Delta n > 2$ transitions feeding the n = 3 levels. All other radiative transition rates were calculated using the above charge distribution parameters to determine the wave functions and energies needed in Eq(1). Only E1 transitions were assumed from levels with n > 4. Non-radiative transitions other than Auger transitions were ignored[12].

In Table I we show a comparison of predicted and observed energies and intensities (corrected for relative detector efficiency) of 4d→2p and 3p→1s E1 transitions and the nearby 4f→2p and 3d→1s transitions. The 4→2 transitions show excellent agreement between experiment and theory for both the energies and relative intensities. Of course the $4f_{5/2}\to 2p_{3/2}$ transition could not be observed directly since it cannot be resolved from the $4d_{5/2}\to 2p_{3/2}$ which is more than ten times as intense. But the assumption of its presence is consistent with the observed intensity ratios.

For the most part this excellent agreement is also found in the $3\to 1$ transitions. However, there are two anomalies which at present remain unexplained. The $3p_{3/2}\to 1s_{1/2}$ transition seems less intense than expected. If we ignore this transition, we find quite good agreement between experiment and theory for the other intensities. This possibly suggests a robbing of the $3p_{3/2}$ level by some unknown (probably non-radiative) process. The same transitions in ^{206}Pb show no such effect but a preliminary analysis of the $3p_{3/2}\to 2s_{1/2}$ to $3p_{1/2}\to 2s_{1/2}$ intensity ratio in ^{208}Pb is not inconsistent with the present anomaly.

The second anomaly involves the displacement of the 3p doublet relative to the 3d doublet. Table II lists the observed fine-structure splittings in the $3\to 1$ transitions. The observed 3p and 3d splittings agree quite well with theory but the $3p_{3/2}\to 3d_{5/2}$ energy difference is nearly four standard deviations from theory. In contrast to the intensity anomaly mentioned above, this anomaly does persist in the ^{206}Pb data. Since the 3p level was not used to determine the charge distribution given in Table I it is conceivable that this anomaly might disappear if a different functional form for the charge distribution had been assumed. Various forms with faster fall-offs[13] have been tried but have not as yet removed this effect. We are investigating other forms of the charge distribution.

In summary we feel that our data strongly demonstrate the presence of E2 atomic transitions in ^{208}Pb. The importance of taking such transitions into consideration for muonic x-ray data interpretation in terms of nuclear charge distributions cannot be overstressed. These transition energies can nearly coincide with those of inner E1 transitions and thus may confuse the interpretations.

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TABLE CAPTIONS

Table I

Energies and relative intensities of $4 \rightarrow 2$ and $3 \rightarrow 1$ transitions in muonic ^{208}Pb.

Table II

$3p \rightarrow 1s$ and $3d \rightarrow 1s$ fine structure splittings in muonic ^{208}Pb (KeV).

TABLE I

TRANSITION	ENERGY OBSERVED	ENERGY PREDICTED[a]	TRANSITION RATE P(EL) 10^{15} sec	REL. POP. OF INITIAL STATE	TYPE TRANSITION	NO. OF EVENTS OBSERVED	NO. OF EVENTS PREDICTED[c]
$4f_{7/2} \rightarrow 2p_{3/2}$	3438.23±.74	3438.56	4.7	0.408[b]	E2	1371±136	1417
$4f_{5/2} \rightarrow 2p_{1/2}$	3613.02±.84	3613.94	3.3	0.307[b]	E2	641±120	686
$4d_{5/2} \rightarrow 2p_{3/2}$	3429.24±.74	3429.88	173.	0.044[b]	E1	3463±195	3035
$4f_{5/2} \rightarrow 2p_{3/2}$		3429.37	0.9	0.307[b]	E2		193
$4d_{3/2} \rightarrow 2p_{1/2}$	3596.78±.85	3596.73	152.1	0.03	E1	1554±166	1730
$3d_{5/2} \rightarrow 1s_{1/2}$	8465.74±2.0	8463.1	28.1	0.464	E2	1089±80	1157
$3d_{3/2} \rightarrow 1s_{1/2}$	8422.8±2.0	8420.3	26.3	0.316	E2	693±57	670
$3p_{3/2} \rightarrow 1s_{1/2}$	8502.9±2.1	8502.0	320.3	0.020	E1	408±60	688
$3p_{1/2} \rightarrow 1s_{1/2}$	8455.0±2.1	8454.6	229.3	0.010	E1	354±55	309

[a] Assumed two parameter Fermi distribution with $c=1.1234\, A^{1/3}F$, n=12.689 and nuclear polarization $E_{1s_{1/2}}=11.65$ KeV.

[b] Assumed point nucleus wave functions with statistical distribution in n = 14 level.

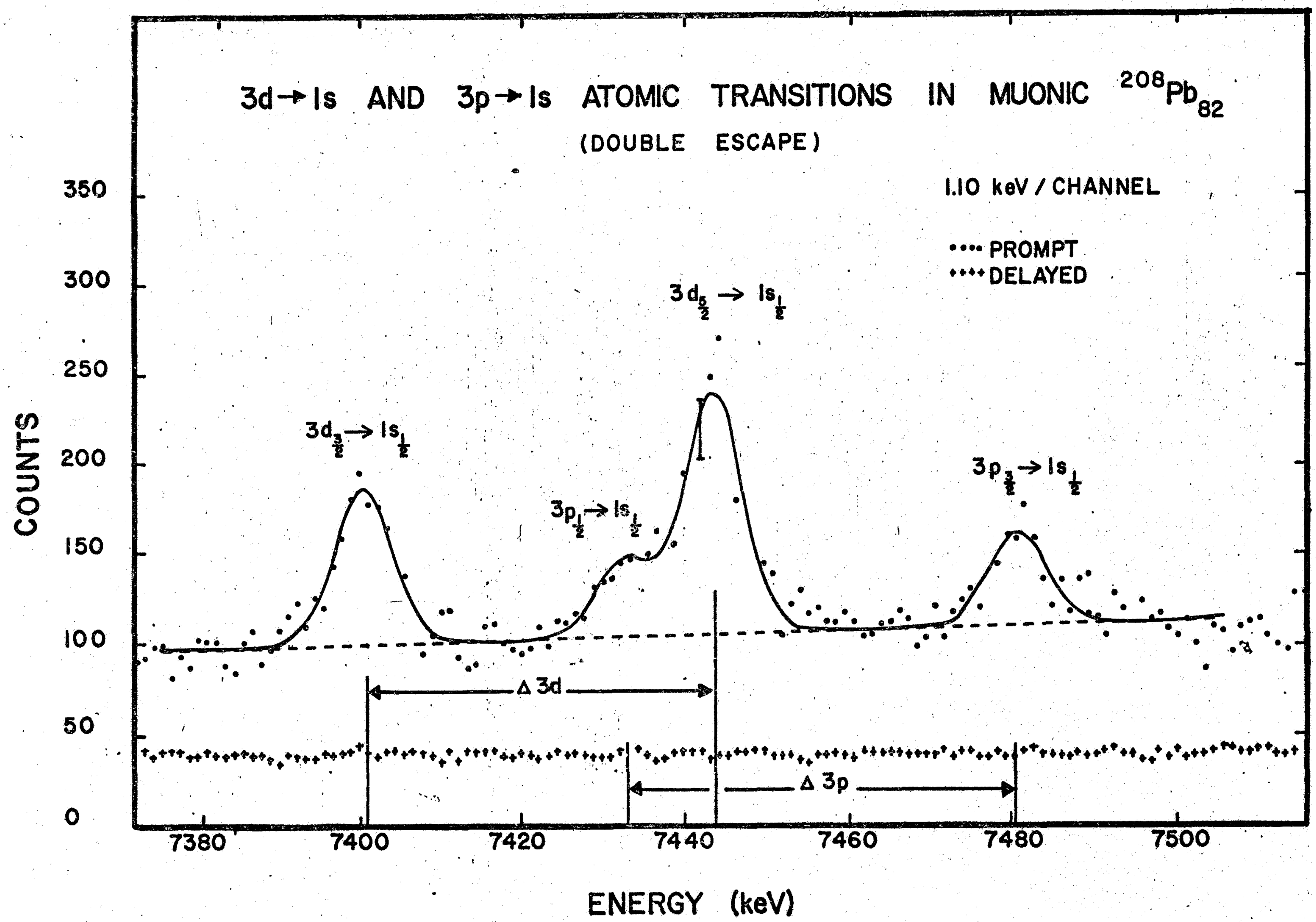
[c] The predicted $4 \rightarrow 2$ intensities are normalized to the observed lines; the predicted $3 \rightarrow 1$ intensities are normalized excluding the $3p_{3/2} \rightarrow 1s_{1/2}$.

TABLE II

LEVELS	ENERGY DIFFERENCE	THEORY
$3d_{5/2} - 3d_{3/2}$	42.84 ± 0.32	42.83
$3p_{3/2} - 3p_{1/2}$	47.56 ± 0.82	47.40
$3p_{3/2} - 3d_{5/2}$	37.05 ± 0.53	38.92

Figure Captions

Figure 1 - ^{208}Pb muonic spectrum showing $3d \rightarrow 1s$ and $3p \rightarrow 1s$ doublets. The dots represent prompt data; the crosses represent delayed data. The heavy curve is a least squares fit of 4 Gaussians and a two parameter exponential background (indicated by the dashed line). The relative intensities of the peaks are indicated by vertical bars located at the centroid of each peak.



3d → 1s AND 3p → 1s ATOMIC TRANSITIONS IN MUONIC $^{208}Pb_{82}$
(DOUBLE ESCAPE)
1.10 keV / CHANNEL
•••• PROMPT
++++ DELAYED
$3d_{3/2} \rightarrow 1s_{1/2}$
$3p_{1/2} \rightarrow 1s_{1/2}$
$3d_{5/2} \rightarrow 1s_{1/2}$
$3p_{3/2} \rightarrow 1s_{1/2}$
Δ 3d
Δ 3p
COUNTS
0
50
100
150
200
250
300
350
7380
7400
7420
7440
7460
7480
7500
ENERGY (keV)